## **Activity 1** Permutations

**Aim:** Use the multiplication principle to calculate number of possible arrangements.

1. Alfred, Blanche, Caleb and Debbie are posing for a group photograph. Complete this list of all 24 possible arrangements for the photo.

| | | | |
|---|---|---|---|
| A B C D | B A C D | C . . . | D . . . |
| A B D C | | | |
| A C . . | | | |
| | | | |
| | | | |
| | | | |

2. The number of arrangements can be calculated as follows: Starting from one end there are four choices. For each of these there are then 3 choices for the person next to them. For each of these a further 2 choices for the next person and one choice for the last person, i.e. $4 \times 3 \times 2 \times 1 = 24$ arrangements.

This can be summarised as

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| 1st choice | 2nd choice | 3rd choice | 4th choice |

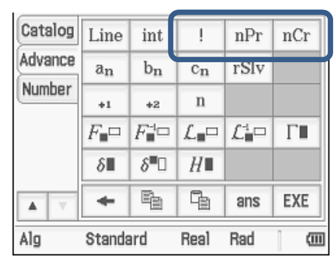
a) If the group is joined by Ernie, how many arrangements are now possible?

b) If the group is joined by Ernie and Frances how many arrangements are now possible?

3. Complete the table using ClassPad to evaluate the factorials.

**Factorial commands**

- Open $\sqrt{\alpha}$ (Main) main
- Press Keyboard
- Tap ▼
- Tap Advance
- Select !




| Expression | 2! | 3! | 4! | 5! | 6! | 7! | | |
|---|---|---|---|---|---|---|---|---|
| Value | | | | | | | 1 | 3628800 |

4. Use factorial notation to express your answers to:

a) Q1, the number of arrangements of 4 people

b) Q2 a) the number of arrangements of 5 people

c) Q2 b) the number of arrangements of 6 people and

d) the number of arrangements of 10 people.

5. The group of friends have single tickets to a number of rides at the show.

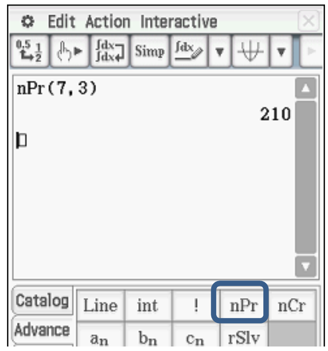
Complete the table to show the number of ways the friends can take the rides. Each friend has no more than 1 ride.

| | | Number of friends | | | |
|---|---|---|---|---|---|
| | | 4 | 5 | 6 | 7 |
| Number of single tickets | 2 | | 20 | | |
| | 3 | | | | |
| | 4 | | | | |
| | 5 | | | | |

6. Complete the table using ClassPad to evaluate the permutations.

**Permutation commands**

- From the Keyboard, Advance tab
- Select nPr
- Enter the number choosing from, then the number selected




| Expression | $P_2^7$ | | $P_3^{10}$ | $P_4^{10}$ | $P_2^n$ | |
|---|---|---|---|---|---|---|
| ClassPad input | nPr(7,2) | nPr(8,2) | | | | |
| Value | | | | | | 90 |

7. How many five-letter code words can be made from the letters in *numbers* when each letter can be:

a) reused?

b) used only once?

8. The Melbourne cup has 26 runners. In how many ways can the first three places be filled?

9. There is space for 8 books on a shelf.
How many arrangements are possible if there are:

a) 8 different books to choose from?

b) 12 different books to choose from?

c) 12 different books to choose from but a particular book is to be put on the left hand end and another is to be put on the right hand end?

## Learning notes

Q1 When listing possibilities it is easier to be systematic. The table provided encourages you to follow a pattern.

Q2 A number of techniques may assist. For example use a box or dash for each selection and then write the number of ways that selection can be filled.

Q3&4 are designed to establish factorial notation as a shorthand way of writing a product of consecutive integers. E.g. $4 \times 3 \times 2 \times 1 = 4!$ (read as 4 factorial).

Q5&6 Investigate problems where not all the people can get a ticket. This leads to using permutations as a shorthand way of writing the number of possible arrangements.

Q8&9 Some problems where using permutations is the most efficient method. Think about the number there are to choose from and how many are selected for a single arrangement. I.e. ${}^{n}P_{r} = \frac{n!}{(n-r)!}$, (select $r$ from $n$)

Permutations can be split into two types: those where repetition is allowed and those where repetition is forbidden. In either case, the order of the arrangement is important which is characteristic of a permutation. See Activity 4 *Combinations* to see problems where order is not important.